647510 **MSC4MatC401x** **Seat No :________**

# M.Sc.(Maths) Semester - 4 (*CBCS*) Examination
# April/May-2022 (NEW COURSE)
# Linear Algebra (CORE)

**Time: 2:30 Hours** **Marks: 70**

**Instructions:**

1. All questions are compulsory.
2. Figures to the right indicate marks.

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**Q.1 (A) Answer any two of the following questions.** **[10]**

(1) Let $V$ be a finite dimensional vector space over $F$ and $T \in A_F(V)$. Show that $T$ is invertible if and only if there exists a vector $\theta \neq v \in V$ such that $T(v) = \theta$.

(2) In standard notations prove that if $T \in A_F(V)$ then $T$ can have at most $n$ distinct characteristic roots in $F$, Where $n = \dim_F V$.

(3) Prove that similarity of linear transformation is an equivalence relation on $A_F(V)$.

**Q.1 (B) Answer any one of the following questions.** **[04]**

(1) Let $T \in A_F(V)$ and $q(x) \in F[x]$. If $\lambda \in F$ is a characteristic root of $T$, then show that $q(\lambda)$ is a characteristic root of $q(T)$.

(2) Let $B = \{v_1, v_2\}$ be a basis of a vector space V. Let $T: V \to V$ be defined as $T(v_1) = v_1 - 2v_2, T(v_2) = v_2 - v_1$. Find Characteristic roots of $T$.

**Q.2 (A) Answer any two of the following questions.** **[10]**

(1) Let $V$ be an $n$ dimensional vector space over $F$ and $T \in A_F(V)$. If $T$ has all its characteristic roots in $F$ then show that $T$ is triangular.

(2) Let $T \in A_F(V)$ be nilpotent. Prove that $T^n = 0$, where $n = \dim_F V$.

(3) State and prove necessary and sufficient condition for two nilpotent linear transformations to be similar.

**Q.2 (B) Answer any one of the following questions.** **[04]**

(1) Write an example of a nilpotent linear transformation $T : R^3 \to R^3$ and show that it is nilpotent.

(2) Let $T : R^3 \to R^3$ be defined as $T(e_1) = (0,1,1), T(e_2) = T(e_3) = (0,0,0)$. Find index of nilpotence of $T$ and the invariants of T.

**Q.3 (A) Answer any two of the following questions.** **[10]**

(1) Let $T \in A_F(V)$ (Where $\dim_F V = n$) be such that $T$ has all its characteristic roots in $F$. Then show that there exists a basis $B$ of $V$ over $F$ such that the matrix of $T$ with respect to the basis $B$ is in Jordan form.

(2) Find Jordan canonical form of the matrix $\begin{pmatrix} 2 & -1 & 1 \\ 1 & 5 & -2 \\ 0 & 1 & 2 \end{pmatrix}$.

(3) State and prove primary decomposition theorem.

**Q.3 (B) Answer any one of the following questions.** **[04]**

(1) Determine the number of possible Jordan canonical forms of a $7 \times 7$ matrix having minimal polynomial $(x-2)^2(x-3)^2$ and characteristic polynomial $(x-2)^4(x-3)^3$.

(2) Define companion matrix and Write companion matrix corresponding to the polynomial $m(x) = x^3 - 4x^2 + 5$.

**Q.4 (A) Answer any two of the following questions.** **[10]**

(1) Let $A, B \in F_n$. Define $tr(A)$ and prove that $tr\,(AB) = tr(BA)$.

(2) State and prove Jacobson's lemma.

(3) Let $(\,V, < >\,)$ be a finite dimensional inner product space over $C$and $T \in A_C(V)$ then show that $T$ is unitary if and only if $|\lambda| = 1$ for each characteristic root $\lambda$ of $T$.

**Q.4 (B) Answer any one of the following questions.** **[04]**

(1) Prove that all characteristic roots of a Hermitian matrix are real.

(2) Let $A \in F_n$be triangular. Show that $\det(A)$is the product of main diagonal entries.

**Q.5 (A) Answer any two of the following questions.** **[10]**

(1) Find rank of the bilinear map $f: R^2 \times R^2 \to R$,

$f\big((x_1, y_1), (x_2, y_2)\big) = x_1x_2 + x_1y_2 + y_1x_2 + \; y_1y_2 \;\; \big((x_1, y_1), (x_2, y_2) \in R^2\big)$.

(2) Let V be an $n$ dimensional vector space over a $C$ and $f: V \;\times V \to C$ be a non-degenerate symmetric bilinear form on $V$. Then show that the group $G = \{\, T \in A(V): T\ preseves\ f \,\}$ is isomorphic to $O(n, C)$.

(3) Determine the rank, index and signature of the quadratic form

$x^2 + 2y^2 - 7z^2 - 4xy + 8xz$.

**Q.5 (B) Answer any one of the following questions.** **[04]**

(1) Define symmetric bilinear form and write an example of it.

(2) Define bilinear form and non-degenerate bilinear map.

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